Kinetic Electron Closures for Electromagnetic Simulation of Drift and Shear-Alfvén Waves

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Kinetic Electron Closures for Electromagnetic Simulation -- Outline

1. A practical algorithm for particle simulation of electromagnetic drift-wave turbulence and transport with kinetic ions and electrons

2. Examples – Simulations of kinetic shear Alfvén waves, and collisionless drift wave and ion-temperature-gradient instabilities at finite β in a two-dimensional unsheared slab

3. Extension to toroidal flux-tube algorithm

4. Summary --

   Successful particle simulations of shear-Alfvén waves and electromagnetic drift-wave and ITG instabilities with kinetic electrons for $\beta m_i / m_e > 1$ (hot core plasmas) in slab. Toroidal code being debugged.

Related work within the SciDAC Plasma Microturbulence Project: continuum methods -- GS2 by W. Dorland, et al., and GYRO by Waltz and Candy; particle methods -- Z. Lin and L. Chen, W. Lee.

How to Accommodate $\beta m_i/m_e$>>1 ? -- Hybrid II Electromagnetic Algorithm

- Extend the "massless" electron hybrid model of Parker, *et al.* and P. Snyder (Ph.D. thesis, Princeton U., 1999) to include drift-kinetic electrons.

- Consider the modified electron momentum equation (Ohm's law) in slab geometry:

$$en_{0e}\vec{E}\ \ \hat{b}^{(0)} = -\ \ _{\|}P_{\|e} + \frac{\delta\vec{B}}{B}\quad en_{0e}\phi - n_{0e}m_e(\partial/\partial t + \vec{v}_{ExB}\quad)u_{\|e}$$

where $\ _{\|}P_{\|e} =\ \ _{\|}P_{\|e}^{(0)} + T_{\|e}^{(0)}\ \ _{\|}\delta n_e^{(0)} + n_{0e}\ \ _{\|}\delta T_{\|e}$ with $\ _{\|}(T_{\|e}^{eq} + \delta T_{\|e}) = 0,\ T_{\|e}^{(0)}$ is a constant, $\delta n_e^{(0)} = \delta n_e - \Delta n_e^K$ =electron fluid density, $\Delta n_e^K = \int d^3 v h_e$ is the *split-weight* δf kinetic increment, and δn_e = total perturbed density consistent with moment of *split-weight* electron distribution function (like Lin and Chen, 2001):

$$f_e = f_M(\vec{x},\vec{v}) + \left(\delta n_e^{(0)} \big/ n_{0e}\right) f_M(\vec{v}) + h_e(\vec{x},\vec{v})$$

- Use Ohm's law to advance $A_{\|}$, $\partial A_{\|} \big/ c\partial t = (\vec{E} + \quad \phi)\ \ \hat{b}^{(0)} = ...$

Hybrid II Electromagnetic Algorithm (cont'd)

- With updated $A_{\|}$ use Ampere's law to determine parallel electron current:

$\Gamma_{\|e} = n_{0e}u_{\|e} = \frac{c^2}{4\pi e} \quad {}^{2}\frac{A_{\|}}{c} + \bar{\Gamma}_{\|i}$, where $\bar{\Gamma}_{\|i}$ is the gyrokinetic parallel ion current.

- Use the electron continuity equation to advance the total electron density:

$$\frac{\partial \delta n_e}{\partial t} + n_{0e}(\vec{B}^{(0)} + \delta\vec{B} \;) \quad \frac{u_{\|e}}{B} + \vec{v}_{E\times B} \quad (n_e^{eq} + \delta n_e) = 0$$

(assumes no magnetic curvature)

- Determine the electric potential ϕ from the quasineutrality relation using the updated electron and gyrokinetic ion densities

- Advance the gyrokinetic ions and the drift-kinetic electrons with same Δt.

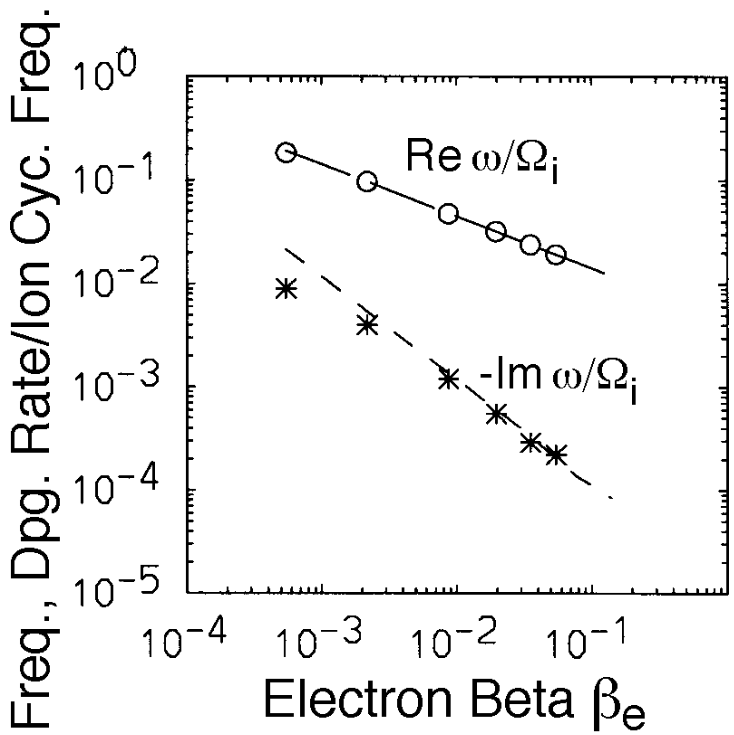
- From drift-kinetic equation for electrons with split-weights (after cancellations),

$$\frac{dw_j^e}{dt} = (\vec{\mathrm{v}}_{ExB} \;\hat{x} + \mathrm{v}_{\|}\frac{\delta B_x}{B_0})\kappa_{Te}(\frac{\mathrm{v}^2}{\mathrm{v}_e^2} - \frac{3}{2}) + \quad {}_{\|}u_{\|e}$$

using $|\,\Delta n_e^K / \delta n_e^{(0)}| << 1$ as an expansion parameter.

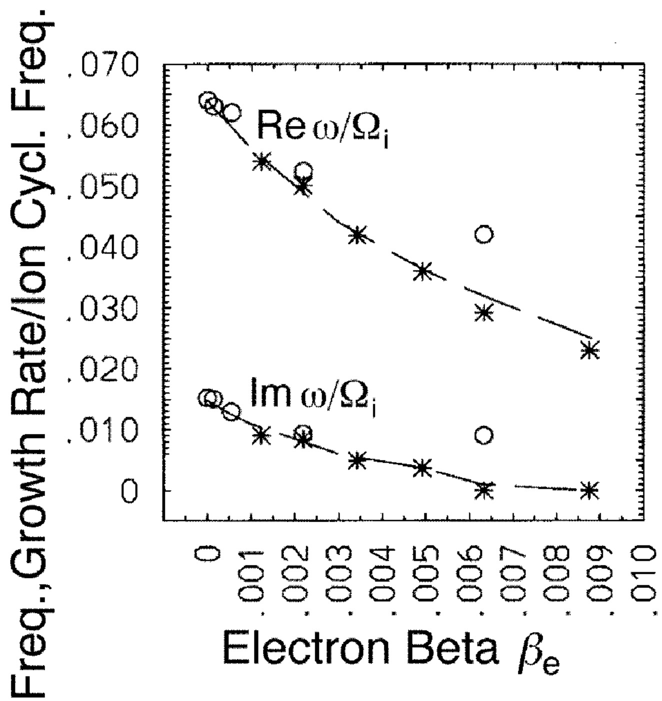
Hybrid II δf Algorithm – Improved Shear Alfvén-Wave Simulations

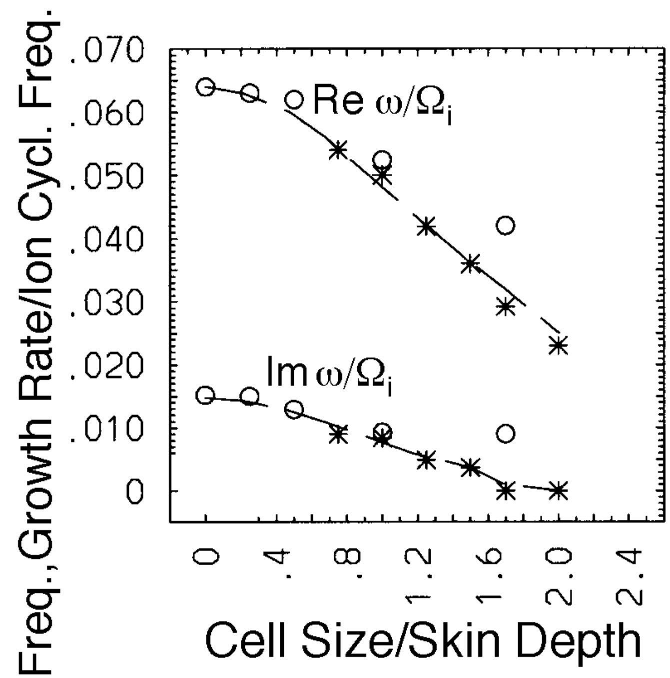
- Simulations of kinetic shear-Alfvén waves in slab.
  Parameters: $k_y\rho_s = \pi/8$, $T_e=T_i$, $B_y/B_0=0.01$, $\rho_s=2\Delta y$, 32x32 grid, (0,1) mode
  theory - - - simulation results: o = $\mathrm{Re}\,\omega / \Omega_i$, $= -\mathrm{Im}\,\omega / \Omega_i$ Landau dpg. rate



- No restriction on $\omega_{pe}\Delta y/c$ and results are similar to Z. Lin and L. Chen's 2001 reported results. (As $\beta_e m_i / m_e \to 0$ the algorithm fails and goes unstable.)

- δ*f* slab simulations of collisionless drift-wave instability with no magnetic shear. Parameters: $k_y\rho_s = \pi/4$, $\rho_s/L_n = 0.2$, $T_e = T_i$, $B_y/B_0 = 0.01$, $\rho_s = 2\Delta y$, 16x16 grid, and (0,1) mode, theory (J. Cummings Ph.D. thesis) - - -,
  o= standard δ*f* simulation, = extended Hybrid II code with kinetic electrons.

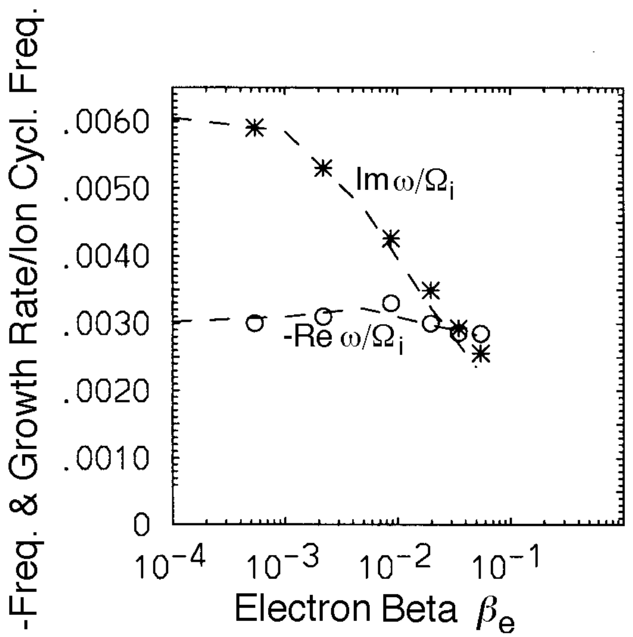




- The Hybrid II algorithm gives good results for $\beta m_i/m_e > 1$ and any skin depth, while the standard δ*f* simulation fails except for $\beta m_i/m_e \leq 1$ and $\Delta y < c/\omega_{pe}$.

Hybrid II Code – Slab ITG Instability Accurate for $\beta m_i/m_e$>1

- 2D Hybrid II simulations of shearless ITG accurate for $\beta m_i/m_e > 1$ and no constraint on the skin depth, i.e., c/ω_{pe} relative to the cell size Δx. Accommodates finite η_e.



- 2D slab simulations with no shear, θ=0.01, $T_e=T_i$, $\eta_i=\eta_e$=4, ρ_s/L_n=0.1, Ω_e/ω_{pe}=1, m_i/m_e=1836, $\rho_s=2\Delta x$, 32×32 grid. Frequency and growth rates for the (0,1) mode ($k_y\rho_s \approx \pi/8$) vs. $\beta_e=(\omega_{pe}\Delta x/c)^2(\rho_s/\Delta x)^2(m_e/m_i)(\omega_{pe}/\Omega_e)^2$ theory (- - -) (J. Cummings, Ph.D. Thesis, 1995) (±10% error bars in obs. Re ω)

Toroidal Flux-tube Implementation of the Hybrid II Algorithm

- Determine the parallel electric field from the modified electron momentum equation (Ohm's law) including toroidicity (ref: P. Snyder and G. Hammett)

$$en_{0e}\vec{E}\ \ \hat{b}^{(0)} = -\ \ {}_{\|}P_{\|e} + \frac{\delta\vec{B}}{B}\ \ \ en_{0e}\phi - n_{0e}m_e(\partial/\partial t + \vec{v}_{ExB}\ \ \ )u_{\|e}$$

$$-\left(\tfrac{1}{2}\delta P\ _e - \delta P_{\|e}\right)\hat{b}^{(0)}\ \ \ \ln B$$

where $\ {}_{\|}P_{\|e} = \ \ {}_{\|}P_{\|e}^{(0)} + T_{\|e}^{(0)}\ \ {}_{\|}(\delta n_e - \delta n_e^K) + n_{0e}\ \ {}_{\|}\delta T_{\|e}$ with $\ {}_{\|}(T_{\|e}^{eq} + \delta T_{\|e}) = 0$.

- Use Ohm's law to advance $A_{\|}$, $\partial A_{\|}/c\partial t = (\vec{E} + \ \ \phi)\ \ \hat{b}^{(0)} = \ldots$

- With the updated $A_{\|}$ use Ampere's law to determine parallel electron flux:

$\Gamma_{\|e} = n_{0e}u_{\|e} = \frac{c^2}{4\pi e}\ \ \ {}^2\ \frac{A_{\|}}{c} + \bar{\Gamma}_{\|i}$, where $\bar{\Gamma}_{\|i}$ is the gyrokinetic parallel ion current.

- Use the electron continuity equation to advance the total electron density:

$$\frac{\partial\delta n_e}{\partial t} + n_{0e}(\vec{B}^{(0)} + \delta\vec{B}\ \ )\ \ \ \frac{u_{\|e}}{B} + \vec{v}_{E\times B}\ \ \ \ n_e$$

$$+\frac{1}{m_e\Omega_e B^2}(\vec{B}\times\ \ \ B)\ \ \ (\tfrac{1}{2}\delta P\ _e + \delta P_{\|e}) + \frac{2n_{0e}}{B^3}(\vec{B}\times\ \ \ B)\ \ \ \ \phi = 0$$

- Determine the electric potential ϕ from the quasineutrality relation using the updated electron and gyrokinetic ion densities:

- Advance the gyrokinetic ions and the drift-kinetic electrons including the toroidal drifts: $\vec{v}_{gs} = v_{\|}\hat{b} + \vec{v}_{E\times B} + \vec{v}_{ds}$, $\quad \vec{v}_{ds} = \frac{v_{\|}^2 + v^2/2}{\Omega_s B^2}\vec{B}\times \quad B$, $\quad \Omega_s \equiv q_s B_0 / m_s c$

  and mirroring.

- From drift-kinetic equation for electrons with split-weights (after cancellations)

$$\frac{d}{dt} w_i^e = (\kappa_e - \kappa_{ne})\hat{x} \ (\vec{v}_{E\times B} + v_{\|}\hat{b}) - \vec{v}_{de} \quad \delta n_e / n_{0e} + \vec{B} \quad (u_{\|e}/B) + (v_{\|}/v_e^2)(\tfrac{\partial}{\partial t} + \vec{v}_{E\times B} \quad)u_{\|e}$$

$$+ v_{\|}(\hat{b}^{(0)} \quad \ln B)(\tfrac{1}{2}\delta p \ \ _e - \delta p_{\|e})/(n_{0e}T_e^{(0)}) + \vec{v}_{E\times B} \ (\varepsilon_{\|}\hat{b} \quad \hat{b} + \tfrac{1}{2}\varepsilon \quad \ln B)/T_e^{(0)}$$

$$+ (n_{0e}m_e\Omega_e B^2)(\vec{B}\times \quad B) \quad (\delta p_{\|e} + \tfrac{1}{2}\delta p \ \ _e) + (2c/B^3)(\vec{B}\times \quad B) \quad \phi$$

- Use flux-tube coordinates: $x=r-r_0$, $y=(r_0/q_0)(q\theta-\zeta)$, $z=q_0R_0\theta$

Initial Comparison Run of Toroidal Hybrid II and GEM Codes

- Toroidal ITG simulations with 32x32x32 grid, $\beta=5\times10^{-4}$, $\eta_i=3.5$, $q_0=1.4$, kinetic electrons and ions, $m_i/m_e=1837$, comparing GEM (conventional low-β code) to toroidal Hybrid II preliminary results. Toroidal Hybrid II is being debugged.